

NAG Toolbox for MATLAB

f07vg

1 Purpose

f07vg estimates the condition number of a real triangular band matrix.

2 Syntax

```
[rcond, info] = f07vg(norm_p, uplo, diag, kd, ab, 'n', n)
```

3 Description

f07vg estimates the condition number of a real triangular band matrix A , in either the 1-norm or the ∞ -norm:

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1 \quad \text{or} \quad \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty.$$

Note that $\kappa_\infty(A) = \kappa_1(A^T)$.

Because the condition number is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of the condition number.

The function computes $\|A\|_1$ or $\|A\|_\infty$ exactly, and uses Higham's implementation of Hager's method (see Higham 1988) to estimate $\|A^{-1}\|_1$ or $\|A^{-1}\|_\infty$.

4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **norm_p** – string

Indicates whether $\kappa_1(A)$ or $\kappa_\infty(A)$ is estimated.

norm_p = '1' or 'O'

$\kappa_1(A)$ is estimated.

norm_p = 'I'

$\kappa_\infty(A)$ is estimated.

Constraint: **norm_p** = '1', 'O' or 'I'.

2: **uplo** – string

Indicates whether A is upper or lower triangular.

uplo = 'U'

A is upper triangular.

uplo = 'L'

A is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

3: **diag** – string

Indicates whether A is a nonunit or unit triangular matrix.

diag = 'N'

A is a nonunit triangular matrix.

diag = 'U'

A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = 'N' or 'U'.

4: **kd** – int32 scalar

k_d , the number of superdiagonals of the matrix A if **uplo** = 'U', or the number of subdiagonals if **uplo** = 'L'.

Constraint: **kd** ≥ 0 .

5: **ab(ldab,*)** – double array

The first dimension of the array **ab** must be at least **kd** + 1

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n triangular band matrix A .

The matrix is stored in rows 1 to $k_d + 1$, more precisely,

if **uplo** = 'U', the elements of the upper triangle of A within the band must be stored with element A_{ij} in **ab**($k_d + 1 + i - j, j$) for $\max(1j - k_d) \leq i \leq j$;
if **uplo** = 'L', the elements of the lower triangle of A within the band must be stored with element A_{ij} in **ab**($1 + i - j, j$) for $j \leq i \leq \min(nj + k_d)$.

If **diag** = 'U', the diagonal elements of A are assumed to be 1, and are not referenced.

5.2 Optional Input Parameters

1: **n** – int32 scalar

Default: The second dimension of the array **ab**.

n , the order of the matrix A .

Constraint: **n** ≥ 0 .

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, work, iwork

5.4 Output Parameters

1: **rcond** – double scalar

An estimate of the reciprocal of the condition number of A . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, A is singular to working precision.

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **norm_p**, 2: **uplo**, 3: **diag**, 4: **n**, 5: **kd**, 6: **ab**, 7: **ldab**, 8: **rcond**, 9: **work**, 10: **iwork**, 11: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to f07vg involves solving a number of systems of linear equations of the form $Ax = b$ or $A^T x = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $2nk$ floating-point operations (assuming $n \gg k$) but takes considerably longer than a call to f07ve with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this function is f07vu.

9 Example

```
norm_p = '1';
uplo = 'L';
diag = 'N';
kd = int32(1);
ab = [-4.16, 4.78, 6.32, 0.16;
      -2.25, 5.86, -4.82, 0];
[rcond, info] = f07vg(norm_p, uplo, diag, kd, ab)
```

```
rcond =
    0.0144
info =
    0
```